OCR
Oxford Cambridge and RSA

Friday 16 June 2017 – Afternoon

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 Solve the differential equation

$$\frac{dy}{dx} + y\cot x = 9\operatorname{cosec} x$$

to find y in terms of x subject to the condition $y = \pi$ when $x = \frac{1}{6}\pi$. **[8]**

2 The group G consists of the set $\{1, 5, 7, 11\}$ combined under multiplication modulo 12.

(i) Draw the group table for G. **[2]**

The group H consists of the set $\{1, 3, 5, 7\}$ combined under multiplication modulo 8.

(ii) Determine whether G and H are isomorphic. **[3]**

3 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 25\sin x.$$ **[8]**

4 A plane Π_1 passes through the points $(1, 2, -1)$, $(2, -3, 1)$ and $(-1, 0, 2)$.

(i) Show that the plane Π_1 has equation $11x + 7y + 12z = 13$. **[4]**

The plane Π_2 has equation $3x + y + z = 4$.

(ii) Find a vector equation of the line of intersection of Π_1 and Π_2. **[4]**

(iii) Find the acute angle between Π_1 and Π_2. **[2]**

5 In an Argand diagram the points O, A and B are represented by the complex numbers 0, z and $2e^{\frac{1}{3}\pi i}z$ respectively, where z is a complex number with modulus 5.

(i) Calculate the exact area of the triangle OAB. **[3]**

The numbers $-1 + i$ and $3 + 3i$ are represented by the points P and Q respectively. The complex number w is represented by the point R, such that $PQ = PR$ and angle $QPR = \frac{1}{4}\pi$.

(ii) Sketch an Argand diagram showing P, Q and the two possible positions of R. Calculate the possible values of w, giving your answers in the form $a + bi$. **[5]**

6 The plane Π and the line l have equations

$$\mathbf{r}.\begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} = 7 \text{ and } \mathbf{r} = \lambda\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

respectively. The point A has coordinates $(1, 2, -4)$.

(i) Find the shortest distance from the point A to the plane Π. **[3]**

(ii) Find the acute angle between Π and l. **[3]**

(iii) Find the point where the line parallel to l passing through A intersects the plane Π. **[4]**

7 **(i)** By expressing $\cos\theta$ in terms of $e^{i\theta}$ show that

$$\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10).$$ **[4]**

(ii) Hence solve, for $0 \leqslant \theta \leqslant \pi$,

$$\cos 6\theta + 6\cos 4\theta + 2\cos 2\theta = 3.$$ **[5]**

8 A group G has the elements $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ where $a, b \in \{1, -1, i, -i\}$. The group operation is matrix multiplication. The subset H consists of the matrices with $a = 1$.

(i) State the order of G. **[1]**

(ii) Show that H is a subgroup of G. **[3]**

K is a proper subgroup of G such that H is a proper subgroup of K.

(iii) Show that K must have order 8. **[4]**

(iv) Show that there is only one such subgroup K and identify its elements. **[6]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			$(I =) \exp(\int \cot x \, dx)$ $= e^{\ln \sin x}$ $= \sin x$ $\frac{d}{dx}(y \sin x) = 9$ $y \sin x = 9x + A$ $x = \frac{1}{6}\pi, y = \pi \Rightarrow \frac{1}{2}\pi = \frac{3}{2}\pi + A \Rightarrow A = -\pi$ $y = (9x - \pi) \operatorname{cosec} x$	M1 M1 A1 M1* A1 M1 *M1dep A1 [8]	 Multiply and integrate Correct substitution of given point and constant evaluated Rearrange to isolate "*y*" oe	Must have "y ="
2	(i)		<table><tr><td></td><td>1</td><td>5</td><td>7</td><td>11</td></tr><tr><td>1</td><td>1</td><td>5</td><td>7</td><td>11</td></tr><tr><td>5</td><td>5</td><td>1</td><td>11</td><td>7</td></tr><tr><td>7</td><td>7</td><td>11</td><td>1</td><td>5</td></tr><tr><td>11</td><td>11</td><td>7</td><td>5</td><td>1</td></tr></table>	B1 B1 [2]	Twelve entries correct All correct	
	(ii)		$3.3 = 5.5 = 7.7 = 1$ both groups non-cyclic so isomorphic <u>as only two groups of order 4</u>	M1 M1 A1 [3]	Can be seen in table Or give order of each element (condone omission of e) Or all elements in each group are self-inverse or all have corresponding orders (shown) Can use " $\cong$" So isomorphic as both are V or K_4 or Klein (four-)group or the four-group	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td></td><td></td><td></td><td>ALT

Table is:
<table><tr><td></td><td>1</td><td>3</td><td>5</td><td>7</td></tr><tr><td>1</td><td>1</td><td>3</td><td>5</td><td>7</td></tr><tr><td>3</td><td>3</td><td>1</td><td>7</td><td>5</td></tr><tr><td>5</td><td>5</td><td>7</td><td>1</td><td>3</td></tr><tr><td>7</td><td>7</td><td>5</td><td>3</td><td>1</td></tr></table></td><td>M1</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>Isomorphism:
$1 \leftrightarrow 1, (3,5,7) \leftrightarrow$ any permutation of $(5,7,11)$ or states that structure is same</td><td>M1</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>… so isomorphic</td><td>A1</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[3]</td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td>AE: $\lambda^2 + 6\lambda + 9 = 0$
$\lambda = -3$ (repeated)
CF: $(A + Bx)e^{-3x}$
PI: $y = a\cos x + b\sin x$
$y' = -a\sin x + b\cos x$
$y'' = -a\cos x - b\sin x$
In DE:
$-a\cos x - b\sin x + 6(-a\sin x + b\cos x) + 9(a\cos x + b\sin x) = 25\sin x$

$-a + 6b + 9a = 0$
$-b - 6a + 9b = 25$
$a = -1.5, b = 2$
GS: $y = 2\sin x - 1.5\cos x + (A + Bx)e^{-3x}$</td><td>M1
A1
A1ft
B1

M1

M1

A1
A1

[8]</td><td>

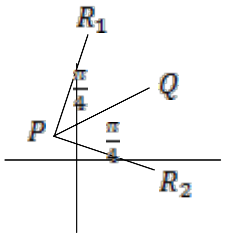
CF for their roots (with two constants)

Differentiate twice and substitute

Compare coefficients

PI correct</td><td></td></tr>
</table>

Question			Answer	Marks	Guidance	
4	(i)		$\overrightarrow{AB} = \begin{pmatrix}2\\-3\\1\end{pmatrix} - \begin{pmatrix}1\\2\\-1\end{pmatrix} = \begin{pmatrix}1\\-5\\2\end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix}-1\\0\\2\end{pmatrix} - \begin{pmatrix}1\\2\\-1\end{pmatrix} = \begin{pmatrix}-2\\-2\\3\end{pmatrix}$	M1*	Any two vectors in plane	Third is $\begin{pmatrix}-1\\0\\2\end{pmatrix} - \begin{pmatrix}2\\-3\\1\end{pmatrix} = \begin{pmatrix}-3\\3\\1\end{pmatrix}$
			$\mathbf{n} = \begin{pmatrix}1\\-5\\2\end{pmatrix} \times \begin{pmatrix}-2\\-2\\3\end{pmatrix} = \begin{pmatrix}-11\\-7\\-12\end{pmatrix} = -\begin{pmatrix}11\\7\\12\end{pmatrix}$	*M1dep	Depends on using attempted vectors in plane Condone 1 incorrect element if no working.	**ALT** $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b} + t\boldsymbol{c}$ Then eliminates one parameter to form 2 equations
				A1	Any multiple – linked to second M1 only Condone omission of final minus sign in this argument	
			$11x + 7y + 12z = 11(1) + 7(2) + 12(-1)$ $11x + 7y + 12z = 13$	A1 [4]	Must show substitution or dot product www. Shown **ag.** Must have some reasoning e.g. AB and AC referenced or described as a vector in the plane, normal referenced**, r = a** + s**b** +t**c**	Then eliminates t to get plane (A2, with A1 awarded for each side of equation **SC4** or verifying that all three points lie on the given plane **and** checking for non-collinearity
	(ii)		$\begin{pmatrix}11\\7\\12\end{pmatrix} \times \begin{pmatrix}3\\1\\1\end{pmatrix} = \begin{pmatrix}-5\\25\\-10\end{pmatrix} = -5\begin{pmatrix}1\\-5\\2\end{pmatrix}$	M1 A1	Attempts cross product of correct vectors Any multiple	
			$x = 0 \Rightarrow y = 7, z = -3$	B1	Find a point on line	or $\begin{pmatrix}\frac{7}{5}\\0\\-\frac{1}{5}\end{pmatrix}$, or $\begin{pmatrix}\frac{3}{2}\\-\frac{1}{2}\\0\end{pmatrix}$, $\begin{pmatrix}1\\2\\-1\end{pmatrix}$, $\begin{pmatrix}2\\-3\\1\end{pmatrix}$
			$\mathbf{r} = \begin{pmatrix}0\\7\\-3\end{pmatrix} + \lambda\begin{pmatrix}1\\-5\\2\end{pmatrix}$	A1 [4]	Oe vector equation **ALT 1:** Find a point on line M1 Find a second point and use to find direction of line M1, A1 Write equation A1	**A2:** Reduce 2 equations to single equation in 2 variables.M1 Write these 2 variables using a parameter. M1 Find third variable parametrically. A1 Write equation. A1

Question		Answer	Marks	Guidance	
	(iii)	$\cos\theta = \dfrac{\left\lvert \begin{pmatrix}11\\7\\12\end{pmatrix}\cdot\begin{pmatrix}3\\1\\1\end{pmatrix}\right\rvert}{\sqrt{11^2+7^2+12^2}\sqrt{3^2+1^2+1^2}}$ $\theta = 0.485$ (or $27.8°$)	M1 A1 [2]		0/2 for $90-\theta$
5	(i)	$\lvert 2e^{\pi i/3}z\rvert = 2\lvert z\rvert = 10$ Area $= \frac{1}{2}\cdot 10\cdot 5\cdot \sin\frac{1}{3}\pi$ $= \frac{25}{2}\sqrt{3}$	B1 M1 A1 [3]	Or $\lvert 2e^{\pi i/3}\rvert = 2$ and scale area at end Use of formula with correct angle	Soi by argand diagram Or 1/2bh since right angled triangle (21.7 inexact)
	(ii)	 $w = -1+i+(4+2i)e^{\pm i\pi/4}$ $= \sqrt{2}-1+(3\sqrt{2}+1)i$ or $3\sqrt{2}-1+(1-\sqrt{2})i$	M1 A1 M1 A1 A1 [5]	Argand diagram with *P*, *Q* and attempt at one *R* at approximately $\frac{\pi}{4}$ to PQ Diagram all correct **SC1** if zero scored out of final 3 marks, for $(4+2i)e^{\pm i\pi/4} = \sqrt{2}+3\sqrt{2}i$ or $3\sqrt{2}-\sqrt{2}i$	Including points labelled, angles labelled or R's in correct quadrant. Distances of Q and R's from P appear equal and gradients approximately correct condone omission of ± at M1 stage 0.41 + 5.24i 3.24 – 0.41i
6	(i)	$\mathbf{r}\cdot\begin{pmatrix}2\\-3\\5\end{pmatrix} = \begin{pmatrix}1\\2\\-4\end{pmatrix}\cdot\begin{pmatrix}2\\-3\\5\end{pmatrix} = -24$ distance $\frac{7--24}{\sqrt{2^2+3^2+5^2}}$ $= \frac{31}{\sqrt{38}}$	M1 M1 A1 [3]	// plane through *A* Oe such as 5.03	**ALT.** $2(1+2\lambda)-3(2-3\lambda)+5(-4+5\lambda)=7$. $\lambda = \frac{31}{38}$ distance = $\sqrt{(2\times\frac{31}{38})^2+(3\times\frac{31}{38})^2+(5\times\frac{31}{38})^2}$

Question			Answer	Marks	Guidance	
	(ii)		$\sin\theta = \frac{\left\lvert\begin{pmatrix}2\\-3\\5\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\2\end{pmatrix}\right\rvert}{\sqrt{2^2+3^2+5^2}\sqrt{1^2+1^2+2^2}}$	M1 M1	For RHS Suitable method for finding required angle	0.1150
			$\theta = 1.46$ (or $83.4°$)	A1 [3]		
	(iii)		$\mathbf{r} = \begin{pmatrix}1\\2\\-4\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\2\end{pmatrix}$	B1		
			$2(1+\lambda) - 3(2-\lambda) + 5(-4+2\lambda) = 7$	M1	Substitute in plane equation	
			$\lambda = \frac{31}{15}$	A1		
			Intersect at $\left(\frac{46}{15}, -\frac{1}{15}, \frac{2}{15}\right)$	A1 [4]	Or position vector. Accept $(3.07, -0.0667, 0.133)$	
7	(i)		$2\cos\theta = e^{i\theta} + e^{-i\theta}$ $2^6\cos^6\theta = e^{6i\theta} + 6e^{4i\theta} + 15e^{2i\theta} + 20 + 15e^{-2i\theta} + 6e^{-4i\theta} + e^{-6i\theta}$	M1 A1	Expand $\left(e^{i\theta} + e^{-i\theta}\right)^6$	
			$2^6\cos^6\theta = \left(e^{6i\theta} + e^{-6i\theta}\right) +$ $\left(6e^{4i\theta} + 6e^{-4i\theta}\right) + \left(15e^{2i\theta} + 15e^{-2i\theta}\right) + 20$ $\Rightarrow 64\cos^6\theta = 2\cos 6\theta + 6(2\cos 4\theta) + 15(2\cos 2\theta) + 20$	M1	for converting to multiple angles	
			$\Rightarrow$ result	A1 [4]	Complete argument including pairing up of e.g. terms in z^4 and z^{-4}	Must equate
	(ii)		$\cos 6\theta + 6\cos 4\theta + 2\cos 2\theta = 3$ $\Rightarrow \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10 = 3 + 13\cos 2\theta + 10$	M1*	Use result from (i)	
			$\Rightarrow 32\cos^6\theta = 13(1 + \cos 2\theta)$	A1	Oe simplified form	
			$\Rightarrow 32\cos^6\theta = 13(2\cos^2\theta)$	*M1dep	Use double angle identity	
			$\Rightarrow \cos\theta = 0$ or $\cos^4\theta = \frac{13}{16}$	A1		
			$\theta = \frac{1}{2}\pi, 0.319, 2.82$	A1 [5]		

Question			Answer	Marks	Guidance	
8	(i)		16	B1 [1]		
	(ii)		$\begin{pmatrix}1 & 0\\0 & b\end{pmatrix}\begin{pmatrix}1 & 0\\0 & c\end{pmatrix}=\begin{pmatrix}1 & 0\\0 & bc\end{pmatrix}\in H$ so closed	B1		If three items dealt with as in scheme, but fail to say "in *H*" then deduct one mark. Must conclude to gain all 3 marks.
			$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\in H$ so contains identity	B1		Must conclude and not address commutativity to gain all 3 marks.
			$\begin{pmatrix}1 & 0\\0 & b\end{pmatrix}^{-1}=\begin{pmatrix}1 & 0\\0 & b^{-1}\end{pmatrix}\in H$ so contains inverses	B1		
			so is (sub) group	[3]		
	(iii)		$\|K\|$ is a factor of their "16"	M1	Use of Lagrange	
			$\|H\|=4$ so 4 is a factor of $\|K\|$	M1	or $\|K\|\geq 4$, if 1st M1 awarded	
			so $\|K\|=4, 8 \text{ or } 16$	A1	May be implied	
			proper subgroups so proper factors so $\|K\|=8$	A1	Complete argument.	
				[4]		

	(iv)		Identifies correct subgroup If $\begin{pmatrix} i & 0 \\ 0 & b \end{pmatrix} \in K$ then $\begin{pmatrix} i & 0 \\ 0 & b \end{pmatrix}^2 = \begin{pmatrix} -1 & 0 \\ 0 & b^2 \end{pmatrix} \in K$ If $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \in K$ for some b then multiplying by elements of H gives $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ for all b But this gives more than 8 elements So $\begin{pmatrix} i & 0 \\ 0 & b \end{pmatrix} \notin K$ Similarly $\begin{pmatrix} -i & 0 \\ 0 & b \end{pmatrix} \notin K$ so $K = \left\{ \begin{pmatrix} \pm 1 & 0 \\ 0 & b \end{pmatrix} : b^4 = 1 \right\}$	B1 M1 M1 A1 M1dep A1 [6]	 Considers $a = i$ or $-i$ with aim to reject it Dep on both previous M marks being gained For full argument	At any stage in solution Possibly in isolation from matrix
			Total	**72**		